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A Comparison of Methods for Determining Optimum Paths in the Problem of Bolza

Faulkner, F.D.; Seibel, E.W.

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$$\int_V f(\mathbf{x})\, d\mathbf{x} \cong \prod_{i=1}^{s} h^{(i)} \left\{ \frac{\sqrt{3}}{6} \sum_{j=1}^{s} [f(\mathbf{x}_{2j-1}) - f(\mathbf{x}_{2j})] + f(\mathbf{x}_{2s+1}) \right\}$$

is exact for polynomials of degree 2 or less, where V is the s-dimensional hyper-rectangle $r^{(i)} - \frac{1}{2}h^{(i)} \leqq x^{(i)} \leqq r^{(i)} + \frac{1}{2}h^{(i)}$ and

$$x_{2j-1}^{(i)} = r^{(i)} + \frac{h^{(i)}}{2} \left[\frac{\sqrt{3}}{6} (1 - \delta_{ij}) - \delta_{ij} \right]$$

$$x_{2j}^{(i)} = r^{(i)} + \frac{h^{(i)}}{2} \left[\frac{\sqrt{3}}{6} (1 - \delta_{ij}) + \delta_{ij} \right]$$

$$x_{2s+1}^{(i)} = r^{(i)} + \frac{\sqrt{3}}{6} h^{(i)}.$$

When this formula is used for subdivisions of a larger region, the contributions from the summation cancel on all interior boundaries, leaving only one point to be evaluated for each interior subregion. Errors for the formula and numerical comparisons with other methods will be given.

2B.4: Computational Procedure for the Calculation of the Subdivisions of the Components of the Analysis of Variance. M. C. Miller, III, *University of Oklahoma Medical School, Oklahoma City, Okla.*

A computational procedure for the calculation of the subdivisions of the components of the analysis of variance is given. The method is essentially one of matrix operations and is shown to be quite general in its applicability. Special emphasis is placed on the use of the method as a means of obtaining the linear, quadratic, etc., components of quantitative main effects and the linear by linear, linear by quadratic, etc., components of the interactions of quantitative by quantitative factors. Also discussed, is the breakdown of the sum of squares of the interaction of qualitative by quantitative factors.

2B.5: A Fast Direct Solution of Poisson's Equation Using Fourier Analysis. R. Hockney and O. Buneman, *Stanford University, Stanford, Calif.*

The motion of interacting charged particles in two dimensions (*e.g.*, in plasmas, electron tubes and ion guns) may be simulated on a computer by a time-stepping process. To do this it is necessary to solve Poisson's equation on a mesh of several thousand nodes in a few seconds. A direct method of solution has been developed involving Fourier analysis in one dimension and Gaussian elimination in the other that solves Poisson's equation on a 48×48 mesh in 2.75 seconds (IBM 7090). The maximum observed error in the potential for a random charge distribution is $\sim 5 \times 10^{-7}$ of the maximum potential difference.

3B TUTORIAL

3B.1: On the Construction of a Compiler for ALGOL 60. Arthur Evans, Jr., *Carnegie Institute of Technology, Pittsburgh, Pa.*

It is possible to use a computer language, such as Algol, for a variety of programs with no concept of the functions of the compiler used with the machine for which the programs are being written. The compiler becomes in the mind of such a programmer, an integral part of the machine. More flexibility in programming and wiser use of computing machinery can be achieved if one has some knowledge of compilers and, in some cases, machine language. This tutorial is presented with the intent of providing a stronger background to the programmer, showing the intricacies of the compiler and mechanism whereby the object program is derived in machine language.

4A PANEL ON INFORMATION RETRIEVAL

Moderator: Louis C. Ray, *System Development Corp., Santa Monica, Calif.*

Panelists: Jack Belzer, *Western Reserve University, Cleveland, Ohio;* John C. Costello, Jr., *Battelle Memorial Institute, Columbus, Ohio;* and Ronald E. Wyllys, *System Development Corp., Santa Monica, Calif.*

The panel will survey current approaches to developing a theoretical base for Information Retrieval work. Special attention will be given to underdeveloped areas of the subject and to pragmatic approaches to the design of IR systems. Audience discussion will be invited.

4B NUMERICAL ANALYSIS II

4B.1: A Comparison of Methods for Determining Optimum Paths in the Problem of Bolza. F. D. Faulkner and E. W. Seibel, *United States Naval Postgraduate School, Monterey, Calif.*

A comparison is made of two methods of determining optimum trajectories. The methods are (1) the differential method due to Bliss, using only extremals and (2) the method of steepest descent, based on the fundamental lemma of the calculus of variations. The study so far has been confined to low order systems. It indicates the differential method converges more rapidly if it converges. Convergence is a problem and it seems that there are problems where the latter method will converge but the former may not. Optimum correction of perturbed paths is a minor problem if the former method is used. The latter method seems to require less luck or skill in getting started.

4B.2: Interpolatory Iteration Functions. J. F. Traub, *Bell Telephone Laboratories, Murray Hill, N. J.*

Let (x_i) be a sequence of approximants to a zero of a real function f. The x_i are generated by an iteration function (I.F.) which samples $f(x)$ and its derivatives at one or more values of x. An important class of I.F., which includes the most commonly used I.F. and their generalizations, are classified and uniformly studied by considering them as generated from hyperosculatory interpolation of f or its inverse at a number of points. In particular, a rigorous derivation of the order of any interpolatory I.F. will be discussed. The derivation requires no a priori assumptions about the *existence* of an order.

4B.3: Matrix Symmetrization Method for the Algebraic Eigenproblem. J. L. Howland and F. J. Farrell, *University of Ottawa, Ottawa, Ont., Canada*

An arbitrary matrix M is said to be symmetrized by the symmetric matrix B in case the matrix product $A = BM$ is symmetric. When a symmetrizing matrix B is known, the eigenproblem $(M - \lambda I)\ \mathbf{x} = 0$ may be reformulated as $(A - \lambda B)\ \mathbf{x} = 0$, and several methods due to S. H. Crandall, and related to Wielandt's "broken iteration" may be applied. In this paper it is shown that Lanczos' method of minimized iterations may be adapted to the calculation of symmetrizing matrices. Moreover, this method produces the symmetrizing matrix B and the symmetric product A in a coordinate system in which B is diagonal and A is tridiagonal.

4B.4: Rapid Methods of Structural Change Analysis. Bertram Klein, *Hughes Aircraft Co., Culver City, Calif.*

Methods are investigated for determining rapidly the effects of changes in matrix problems of the form $AX = L$ without again solving the entire problem. Special techniques are discussed and examples given for treating the approximate methods. The techniques facilitate the parametric analysis of many complex problems, particularly in the field of structures.

4B.5: On the Numerial Solution of Certain Classes of Boundary Value Problems. L. E. Hulbert and F. W. Niedenfuhr, *Ohio State University, Columbus, Ohio*

The boundary collocation technique is a powerful but little used method for solving boundary value problems. A slight modification of this technique has been used in the computer solution of a variety of boundary value problems involving regions of complicated shapes. A particular advantage of the approach is that the computer program requires only minor modifications to